

NOTE

New Physical Criteria for Using Linear Artificial Viscosity

Artificial viscosity [Q] has been used for over 40 years [1] to simulate numerically the propagation of waves in a discretized continuum. It is composed typically of terms that are quadratic and linear in the gradient of the particle velocity, and "switches" (numerical representations of physical criteria) for turning the terms *on* and *off*. The quadratic term is active only in the region of a sharp discontinuity, e.g., a shock, where the velocity gradient is large, and spreads (smooths) the discontinuity over a few computational zones, or a fixed length. The linear Q was introduced to damp numerical noise, but it is active throughout the mesh. Consequently, it can be very dissipative.

We are interested in simulating the propagation of elastic waves, for which only the linear Q is important. The dispersion and attenuation due to the linear Q can dominate the character of the wave propagation [2]. Therefore, an analytic solution is necessary to assess the quality of various Q -formulations. Blake [3] obtained an analytic solution for the propagation of a spherical wave driven by an exponentially decaying pressure applied to the inside surface of a hollow elastic sphere. This is an ideal problem for comparing different formulations of the linear Q . Its physical characteristics are quite similar to waves generated by underground explosions; thus, the Blake solution is relevant to many real applications.

We will show that Blake's analytic solution can be approximated numerically very well, using a standard tensor linear- Q . The key improvement we make is a new formulation of the switch that turns the Q *on* and *off*, which is in contrast to most of the previous work which has focussed on altering only the functional form of the artificial viscosity. Our improvement requires little additional computational overhead beyond incorporating the tensor linear Q and, thus, is very fast computationally.

The linear Q used in many hydrocodes may be expressed as

$$Q_{ij} = a\eta v_{i,j} = a\eta \left[v'_{i,j} + \frac{1}{3} v_{k,k} \delta_{ij} \right], \quad (1)$$

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where $v_{i,j}$ are components of the velocity gradient ($v_{k,k} = \nabla \cdot \mathbf{v}$), η is a function of the local sound speed, the zone size, and the density, and a is a user-specified multiplicative constant. The right side of Eq. (1) is a decomposition of the viscosity into deviatoric (primed) and scalar components (δ_{ij} is the Kronecker delta). The deviatoric term is ignored often, so that only a scalar Q remains. Although a scalar Q is fine for hydrodynamic waves, the tensor form is more appropriate for damping oscillations in elastic waves, which can have large deviatoric components.

The typical switch used with Eq. (1) is Q_{ij} *on* if $\nabla \cdot \mathbf{v} < 0$, i.e., if the material is compressing volumetrically. Our "modified" formulation of the linear Q , which we denote as Q^m , is obtained by specifying a new switching function for Eq. (1). We write

$$Q_{ij}^m = a\eta v_{i,j} [f_1 + f_2], \quad (2a)$$

$$f_1 \equiv \max[\text{sgn}[-\sigma_{ij} v_{i,j}], 0], \quad (2b)$$

$$f_2 \equiv \max \left[\text{sgn} \left[\frac{\partial^2}{\partial t^2} (-\sigma_{ij} v_{i,j}) \right], 0 \right], \quad (2c)$$

where σ_{ij} are the components of the stress tensor (compressive stresses are negative), and $\text{sgn}(\mathbf{x}) = 1$ if $\mathbf{x} > 0$, -1 if $\mathbf{x} < 0$, or 0 if $\mathbf{x} = 0$. Thus, f_1 and f_2 equal 0 or 1. The scalar $\sigma_{ij} v_{i,j}$ is the power per unit volume, so that f_1 and f_2 have simple physical interpretations. The second temporal partial derivative in f_2 is calculated by differencing stored values of the power per unit volume. Figure 1 shows schematically the hypothetical response of a computational mesh to the passage of a uniaxial strain wave. Particle velocity is plotted as a function of distance. The dashed line is an idealized solution for a propagating step wave (compressive or tensile). The solid line is the oscillatory response that would be expected from a numerical solution without Q . The figure also shows the regions of the wavefront in which f_1 and f_2 would be active initially. f_1 is active for compressed regions that are expanding, or for expanded regions that are compressing. f_1 is activated initially only after the peak of the wave; consequently, it does not damp the initial overshoot. f_2 damps the overshoot, because it is constructed from the second derivative of the power per unit volume, so that it

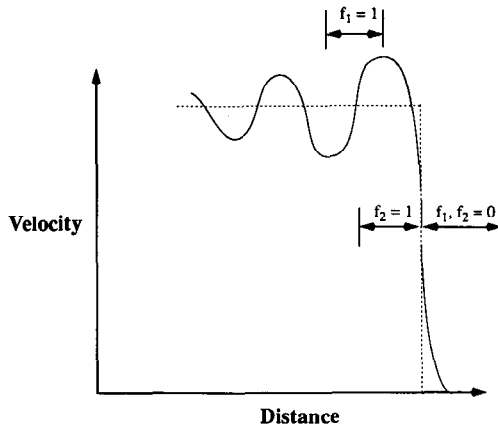


FIG. 1. Schematic response of a computational mesh to the passage of a uniaxial strain wave. The particle velocity is plotted as a function of distance, for the hypothetical numeric (solid line) and exact (dashed line) solutions. The regions are shown where f_1 and f_2 are active initially.

“anticipates” the overshoot. The figure shows that f_2 is activated initially at the midpoint of the wavefront. Although f_2 is defined by a temporal derivative, we have used the approximation $\partial/\partial t \rightarrow c(\partial/\partial x)$, where $c \sim c_L$ (longitudinal sound speed), to compare and contrast in the same figure the regions of initial activity of f_1 and f_2 .

Equations (2) were added to DYNA2D [4, 5], which is a 2D Lagrangian hydrocode. The Blake problem was modelled using a spherical 1D mesh with 100 radial 0.2m thick zones. The driving pressure, P , on the inner surface, which was at a radius of 10m, was $P(\text{GPa}) = 0.1 \exp[-1000t]$, where t is the time in seconds. The density, bulk modulus, and shear modulus of the elastic material were 2000kg/m^3 , $36(\text{GPa})$, and $12.5(\text{GPa})$. The resulting longitudinal sound speed, c_L , is 5.13m/ms . Figures 2–5 show the velocity profile at 3ms for the analytic solution (dashed line) and four formulations of the linear Q (solid line) listed in Table I. Figures 2 and 3 show the results of

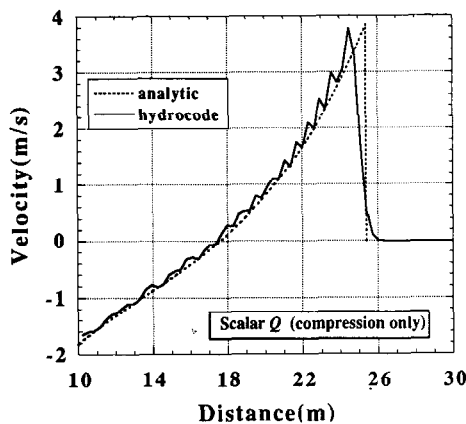


FIG. 2. Velocity as a function of distance at 3ms for the analytic (dashed) and the numeric (solid) solutions. The multiplicative constant, a , is 0.015.

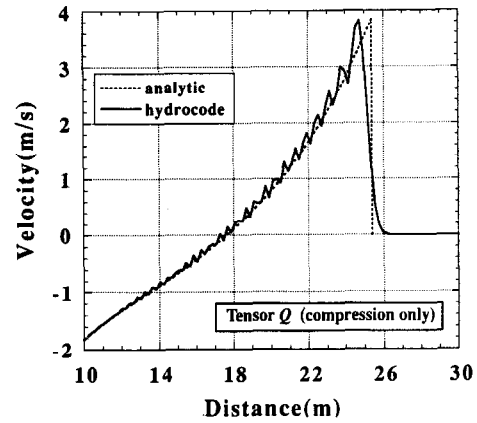


FIG. 3. Velocity as a function of distance at 3ms for the analytic (dashed) and the numeric (solid) solutions. The multiplicative constant, a , is 0.013.

simulations based on variations of Eq. (1). Figure 4 shows results obtained using one formulation of a flux-limited viscosity [6], which replaces the velocity gradient in Eq. (1) with a specially computed velocity difference. Figure 5 shows results obtained using Q^m [Eq. (2)]. For each of the four cases, the value of the multiplicative coefficient, a , was chosen so that the calculated peak velocity would equal the analytic peak velocity. The value of a is larger for Q^m than for the other forms based on Eq. (1), because it is *on* less often than those forms. The functional form of the flux-limited viscosity differs from Eqs. (1), (2), so values of a cannot be compared. When an analytic solution is not available, a is chosen (somewhat subjectively) as small as possible, to minimize oscillations, while maximizing peak amplitudes.

The scalar and tensor viscosities shown in Figs. 2 and 3 do not damp fully the oscillations behind the peak. The numerical results shown in Figs. 2 and 3 are improved slightly if the Q is always *on*, instead of *on* only in compression. However, leaving the Q always *on* introduces too much dissipation, for most realistic problems. This is why

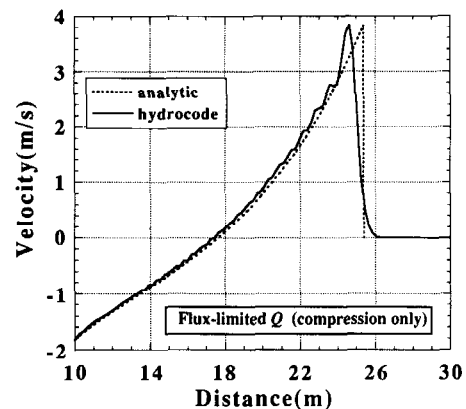


FIG. 4. Velocity as a function of distance at 3ms for the analytic (dashed) and the numeric (solid) solutions. The multiplicative constant, a , is 0.09.

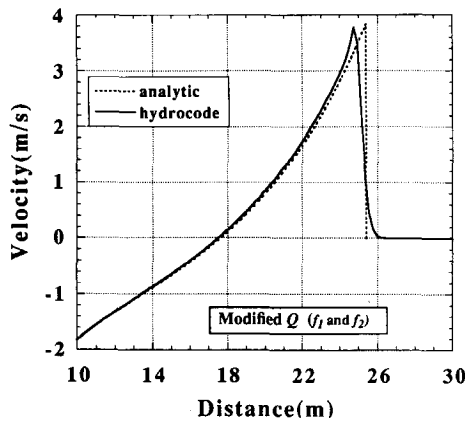


FIG. 5. Velocity as a function of distance at 3ms for the analytic (dashed) and the numeric (solid) solutions. The multiplicative constant, a , is 0.032.

the compression-only condition has become the “industry standard.” Figure 4 shows that the flux-limited viscosity gives better results than those shown in Figs. 2 and 3, but the oscillations are still not damped completely. Figure 5 shows that the solution using the modified Q is nearly free of oscillations and more closely approximates the analytic solution than the other cases. In addition, we examined several other test problems, other forms of Q , and other

TABLE I

Parameters for the Numerical Simulations

Figure	Viscosity type	Activity	a
2	Eq. (1): scalar [$v'_{i,j} = 0$]	Compression only	0.015
3	Eq. (1): tensor	Compression only	0.013
4	Flux-limited [Ref. 6]	Compression only	0.09
5	Eq. (2): modified	f_1 and f_2 (Eqs. (2b), (2c))	0.032

switches. Q^m produced results that exceeded those results in every case.

The purpose of this note was to demonstrate that the accuracy of numerical simulations of wave propagation that use linear artificial viscosity can be improved significantly by altering the traditional activation criteria for the viscosity. Our activation criteria are based on the power per unit volume and can be incorporated easily into 1, 2, and 3D hydrocodes.

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REFERENCES

1. J. von Neumann and R. D. Richtmyer, *J. Appl. Phys.* **21**, 232 (1950).
2. J. A. Viecelli, *J. Comput. Phys.* **12**, 187 (1973).
3. F. G. Blake, Jr., *J. Acoust. Soc. Am.* **24**, 211 (1952).
4. J. O. Hallquist, University of California, Lawrence Livermore National Laboratory Report UCRL-52429, 1978 (unpublished).
5. J. L. Levatin, A. V. Attia, and J. O. Hallquist, University of California, Lawrence Livermore National Laboratory Report UCRL-ID-106104, 1990 (unpublished).
6. D. J. Benson, *Comput. Methods Appl. Mech. Eng.* **93**, 39 (1991).

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